

Classicalize or not to Classicalize?

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Abstract

We show that theories that exhibit classicalization phenomenon cease to do so as soon as they are endowed a Wilsonian weakly-coupled UV-completion that restores perturbative unitarity, despite the fact that such UV-completion does not change the leading structure of the effective low-energy theory. For example, a Chiral Lagrangian of Nambu-Goldstone bosons (pions), with or without the Higgs (QCD) UV-completion looks the same in zero momentum limit, but the latter classicalizes in high energy scattering, whereas the former does not. Thus, theory must make a definite choice, either accept a weakly-coupled UV-completion or be classicalized. The UV-awareness that determines the choice is encoded in sub-leading structure of effective low-energy action. This peculiarity has to do with the fundamental fact that in classicalizing theories high energies correspond to large distances, due to existence of the extended classical configurations sourced by energy. UV-fate of the theory can be parameterized by introducing a concept of a new quantum length-scale, de-classicalization radius. Classicalization is abolished when this radius is a dominant length. We then observe a possibility of a qualitatively new regime, in which a theory classicalizes only within a finite window of energies. We suggest that one possible interpretation of physics above the classicality window is in terms of a quantum theory of unstable extended objects. In order to gain a physical intuition about the possible meaning of such regime, we establish analogy with QCD-type theory (in which all quarks are heavy) which contains breakable QCD-type flux tubes. Extrapolating this analogy to light quark case, we observe that QCD with light quarks can be regarded as a limit of a would-be classicalizing theory in which classicality window collapses to a single scale.

1 Introduction

The focus of the present work is UV-completion of non-renormalizable theories in which interaction strength among light fields is controlled by positive powers of a certain fundamental length, L_* . Such interactions violate perturbative unitarity at energies above L_*^{-1} . The standard (Wilsonian) approach to such theories is, that

they can only be viewed as an effective description of more fundamental weakly-coupled quantum theory valid at distances shorter than L_* . In this picture a weakly-coupled new physics opens up at some intermediate distance m^{-1} , and restores perturbative unitarity.

It was suggested recently [1, 2], that for a class of non-renormalizable theories, the picture of UV-completion can be fundamentally different. Namely, in such theories no weakly-coupled new physics comes into the game. Instead, at high-energies the system unitarizes itself by its own resources, via *classicalization*. That is, in such theories high-energy scattering amplitudes are dominated by production of extended classical configurations, *classicalons*. As a result, hard $2 \rightarrow 2$ scattering processes are exponentially suppressed, and the scattering is dominated by

$$2 \rightarrow \text{something classical} \rightarrow \text{many} . \quad (1)$$

The essential feature of such theories is the existence of a Bosonic degree of freedom, ϕ , a so-called *classicalizer*, which is (self)sourced by energy. Then, at high enough center of mass energy, $\sqrt{s} \gg L_*^{-1}$, the scattering proceeds through the formation of an extended classical configuration of ϕ of size $r_* \gg L_*$. Inevitability of such a configuration is due to the fact that ϕ is sourced by energy. The effective size of it sets the range of the interaction, and thus the cross section as,

$$\sigma \sim r_*^2 . \quad (2)$$

For better understanding of classicalization phenomenon it is useful to adopt an universal definition of the r_* -radius, given in [2], which is equally applicable to classicalizing as well as to non-classicalizing theories. According to this definition, r_* can be viewed as the length that at a given energy sets an effective range of the interaction. That is, r_* is a distance down to which wave-packets propagate essentially freely, without experiencing a significant scattering. In other words, the equation (2) can be used as definition of r_* in an arbitrary theory. Notice, that with this definition r_* is automatically a classical scale, not containing any powers of \hbar in it. Notice also, that such a radius can be defined for an arbitrary quantum system, a familiar example being a so-called classical radius of electron, $r_* = r_e \equiv e^2/m_e$, that controls the interaction range for Thomson scattering.

But, if r_* can be universally-defined, what determines classicalization of a given theory?

In the above language the answer is very simple. The defining property is how the classical length r_* relates to the relevant quantum length-scales of the theory, and how this relation evolves with energy. In weakly-coupled theories r_* is always much smaller than the quantum length-scales, and with increasing \sqrt{s} shrinks and becomes less and less relevant [2]. For example, in Thomson scattering r_* is shorter than the electron Compton wave-length, $r_* \ll m_e^{-1}$. Because of this, by the time

one is able to probe the distance $r_* = r_e$, the quantum effects become important. Such systems are governed by quantum dynamics.

The condition for classicalization is the growth of r_* with energy, in such a way, that above some critical energy it exceeds all the relevant quantum length scales in the system. When this is the case, system classicalizes. This property is exhibited by derivatively -(self)coupled theories, in which ϕ is sourced by energy. In such theories, depending on the concrete interaction picture, r_* typically grows as a positive power of \sqrt{s} ,

$$r_* \sim L_*(L_*\sqrt{s})^\alpha, \quad (3)$$

with $\alpha > 0$. The precise value of α is determined by a leading operator responsible for energy-sourcing, and is model-dependent.

In this note we wish to further deepen the confrontation between the weakly-coupled and classicalizing theories, and show that the latter can be understood as a deformation of the former in which the unitarizing weakly-coupled physics is removed, whereas the leading structure of high-dimensional derivative-couplings of low energy effective theory is kept in tact.

This transition from quantum-weakly-coupled to classicalizing behavior demonstrates explicitly that classicalization is a "self-defense" of a theory when it is deprived of a weakly-coupled UV-completion. Correspondingly, by integrating-in a weakly-coupled physics that restores perturbative unitarity we *de-classicalized* the theory.

We shall arrive to this conclusion by addressing the following (seeming) puzzle. Consider a derivatively-self-coupled low energy theory, which above a scale $m < M_* \equiv L_*^{-1}$ admits a weakly-coupled completion. The role of such theory, for example, can be played by a chiral Lagrangian of a massless Goldstone field, ϕ , which at high energies gets completed as a theory of a weakly-coupled scalar field (Higgs) that spontaneously breaks a continuous global symmetry.

Integrating out a radial (Higgs) degree of freedom, we get an effective low energy theory which contains derivative interactions of the Nambu-Goldstone bosons. As was argued in [1, 2], such a theory classicalizes at a distance $r_* \gg L_*$. Notice, that classicalization is a high-energy, but *long-distance* effect. But, at long distances, seemingly, theory should know nothing about its perturbative UV-completion. Thus, theory should classicalize regardless of the existence of such a completion. But, this is impossible, since, as viewed from high-energy perspective theory is a weakly coupled theory of a scalar, which cannot exhibit any classicalization. Weakly-coupled theories cannot classicalize. Since the two UV-behaviors are very different, one is left with a puzzle.

How can an effective low-energy theory decide which UV-path to choose: *Classicalize or not to Classicalize*?

In this note we shall resolve this seeming puzzle and show, that classicalization and weakly-coupled completions are inter-exclusive. That is, as long as asymptotic

degrees of freedom are fixed, in any given energy range, theory either classicalizes or has a weakly-coupled UV-completion, but not both properties simultaneously.

(Notice, that this statement does not exclude possible duality-type relation between the classicalizing and weakly-coupled descriptions, which changes the definitions of asymptotic states. Such possibility is not our focus at the moment, but will become later when we go to QCD analogy, see below.)

By integrating-in any weakly-coupled physics that restores unitarity we kill classicalization, and theory de-classicalizes. From the first glance, this may come as a surprise, since as argued before, classicalization is a long-distance effect, since it takes place at distances much larger than the cutoff length of the low energy theory, $r_* \gg L_*$. However, the key point is, that in classicalizing theories the long distances are not necessarily equivalent to low-energies, due to existence of extended objects, classicalons, that dominate high-energy scattering. As a result, the scale r_* corresponds to the energies at which the theory is already well-aware of the existence of weakly-coupled unitarizing physics, and classicalons are never formed. The theory de-classicalizes.

As we shall see, the decision about which path to follow is made by the operators that are sub-leading at low-energies, but which carry information about the existence (or non-existence) of unitarity-restoring (weakly-coupled) propagating degrees of freedom in UV. System classicalizes when such information is absent.

Analyzing a possible influence of such "UV-informative" operators on the classicalization dynamics, we shall discover a possibility of qualitatively new regime, in which the system classicalizes within the finite interval of energies. We shall refer to this energy interval as the *window of classicality*. In such theories, at increasing center of mass energy, the system switches from quantum to classical and back to quantum regimes. The space of such theories can be parameterized by introducing a concept of a new quantum length-scale, \bar{r} , which we shall refer to as the de-classicalization radius. This radius encodes information about the importance of the quantum effects on the system and diminishes in the limit $\hbar \rightarrow 0$. In our parameterization, classicality window is determined as the range of energies for which

$$r_* \gg \bar{r}, L_*. \quad (4)$$

Outside the classicality window one of the quantum scales (either L_* or \bar{r}) starts to dominate over r_* , and system becomes quantum. In this parameterization, classicalizing theories correspond to the case when condition (4) is satisfied in arbitrarily deep UV, and classicality window is infinite.

Given the required structure of the low energy theory, the finiteness of classicality window can be detected unambiguously. This rises an interesting question about the physical meaning of theories with finite classicality window. Our analysis shows, that beyond this window in deep-UV theory becomes again quantum, but what are the corresponding degrees of freedom?

We speculate in this direction and suggest, that the UV-theory above classicality window is a quantum theory of unstable extended objects. In order to substantiate

this guess, we use physical intuition coming from QCD-type theories in which all quarks are heavier than the QCD scale, Λ_{QCD} . Such theories contain flux-tubes (QCD-strings) that are practically-stable below certain critical (exponentially-long) size ($\equiv \bar{L}$), whereas the longer tubes are unstable. We show, that for such a system the complete analogs of L_* , r_* and \bar{r} lengths can be defined, in terms of QCD-confinement length ($L_{QCD} \equiv \Lambda_{QCD}^{-1}$), classical length of the QCD flux-tube (L), and the ratio of the latter length-squared to the critical instability length (L^2/\bar{L}), respectively. This analogy however has obvious limitations, since such a QCD is a theory with a mass gap and contains no light analog of the classicalizer field ϕ . In order to create such an analog, in form of pions, we need to push the quark masses below the QCD-scale. But, such a deformation of the theory eliminates the classicality window, since QCD flux-tubes longer than L_{QCD} become unstable. This gives a complementary understanding of why, due to the presence of the heavier resonances, the pion Lagrangian does not classicalize at high energies.

Thus, in classicalization language, the low-energy QCD can be viewed as a (would-be classicalizing) theory in which classicality window collapses towards the QCD-scale.

2 Classicalization and De-Classicalization

Before going to more detailed analysis, let us explain the main reason behind the de-classicalization phenomenon.

For this, let us first briefly summarize the essence of classicalization effect. The latter phenomenon is exhibited by a class of theories with derivative self-interactions. As a simple prototype example, consider a theory of a real scalar field,

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{L_*^4}{4} ((\partial_\mu \phi)^2)^2. \quad (5)$$

This theory is symmetric under the shift by an arbitrary constant c ,

$$\phi \rightarrow \phi + c. \quad (6)$$

As it was shown in [1, 2], this theory classicalizes at $\sqrt{s} \gg 1/L_*$. Namely, at high center of mass energies, the scattering takes place at a macroscopic length-scale determined by the classical radius,

$$r_* \equiv L_*(\sqrt{s}L_*)^{\frac{1}{3}}. \quad (7)$$

In order to have an unified description of classicalizing and non-classicalizing theories, it is useful to adopt the definition of r_* radius given in [2]. According to this definition, in a generic theory the physical meaning of r_* -radius is of a distance at which the scattering of a free wave becomes significant. Assuming that for $r = \infty$ and $t = -\infty$, ϕ starts out as an in-wave ϕ_0 satisfying the free-equation,

$$\square \phi_0 = 0, \quad (8)$$

and representing ϕ as a superposition of free and scattered waves,

$$\phi = \phi_0 + \phi_1, \quad (9)$$

r_* -radius is defined as a characteristic distance at which the correction ϕ_1 to a free-wave becomes significant. That is, r_* -radius is determined from the condition,

$$\phi_0(r_*) \sim \phi_1(r_*). \quad (10)$$

Let us now imagine, that we UV-complete the above theory by a weakly-coupled physics at some scale m . Obviously, in order to restore perturbative unitarity, the scale m at which the new physics sets-in must be sufficiently below the unitarity-violation scale L_*^{-1} . In order to make the argument clean, let us take,

$$m \ll L_*^{-1}. \quad (11)$$

The effect of such weakly-coupled UV-completion on the effective low energy action (5) is expressed in endowing an each additional square of the derivative by a propagator with a massive pole at m^2 ,

$$(\partial_\mu \phi)^2 \rightarrow \frac{m^2}{m^2 + \square} (\partial_\mu \phi)^2, \quad (12)$$

so that the low-energy effective action becomes

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{L_*^4}{4} \left((\partial_\mu \phi)^2 \frac{m^2}{m^2 + \square} (\partial_\nu \phi)^2 \right) + \dots \quad (13)$$

The qualitative reason of why weakly-coupled Wilsonian UV-completion de-classicalizes the theory is now clear. Although, at low energies ($\square \ll m^2$) the action (13) effectively reduces to (5), the difference is substantial at high energies ($\square > m^2$). Above this energy, the mass term in the propagator becomes irrelevant and the interaction becomes suppressed by momentum-squared.

As a result of this suppression, in high center of mass scattering, the self-sourcing by energy switches off at distances much larger than the would-be classicalization radius, and waves scatter without ever reaching the classicalization point. Theory de-classicalizes.

Indeed in the absence of weakly-coupled UV-completion, as in theory (5), at center of mass energy $\sqrt{s} \gg L_*^{-1}$ classicalization would happen at distance $r_* \gg L_*$, for which

$$\square \phi_0^2 \sim L_*^{-4}. \quad (14)$$

Notice, that for any free wave satisfying (8), the above condition is equivalent to

$$(\partial_\mu \phi_0)^2 \sim L_*^{-4}. \quad (15)$$

For classicalization it is absolutely essential that r_* -radius for which the equivalent conditions (14), (15) and (10) are satisfied, is much larger than the fundamental (quantum) length L_* . And in theory (5) this is indeed the case, according to (7).

It is now obvious that in a weakly-coupled completion this situation can never be reached, since the distance at which the self-sourcing switches off is given by the radius \bar{r} at which

$$\square\phi_0^2 \sim m^2\phi_0^2. \quad (16)$$

This gives

$$\bar{r} \sim \sqrt{s}/m^2. \quad (17)$$

Obviously, for $m \ll L_*^{-1}$, we have $\bar{r} \gg r_*$ and thus system never reaches the classicalization radius.

We shall now discuss the above effect in more details.

3 Classicalons

As an example of classicalizing theory, consider again the prototype model of derivatively self-interacting scalar, given by the action (5). In order to see why the theory classicalizes at energies $\sqrt{s} \gg 1/L_*$, let us repeat the steps of [2]. For this we shall study the scattering of waves by analyzing the equation of motion following from (5),

$$\partial^\mu(\partial_\mu\phi(1 + L_*^4(\partial_\nu\phi)^2)) = 0. \quad (18)$$

We shall assume that for $r = \infty$ and $t = -\infty$, ϕ is well-approximated by a spherical wave of characteristic frequency ω and the amplitude $A \sim 1$ given by

$$\phi_0 = \frac{\psi(\omega(r+t))}{r}, \quad (19)$$

which solves the free-field equation of motion (8). Notice, that for small occupation number $A \sim 1$, the energy of the wave is $\sqrt{s} \sim \omega$.

We shall now solve the equation (18) iteratively, by representing the ϕ -field according to (9) as a superposition of a free-wave ϕ_0 and a scattered wave ϕ_1 . We shall treat ϕ_1 as a small correction, and try to understand at what distances it becomes significant. The r_* -radius is then determined by a distance for which, ϕ_1 becomes comparable to ϕ_0 , according to (10). For our purposes we do not need to go beyond this point. In the leading approximation, the equation for the correction to the free wave is,

$$\square\phi_1 = -L_*^4\partial^\mu(\partial_\mu\phi_0(\partial_\nu\phi_0)^2). \quad (20)$$

Taking into the account properties of $\psi(\omega(t+r))$ -wave, for $\omega \gg r$, the leading contribution to the right hand side is,

$$\square\phi_1 = -\frac{L_*^4}{r^5}(2\psi^2\psi'' + 8\psi\psi'^2), \quad (21)$$

where prime denotes the derivative with respect to the argument. For $\omega \gg r^{-1}$ the solution of this equation can be approximated by,

$$\phi_1 \simeq -f(\omega(r+t)) \frac{L_*^4}{6r^4}, \quad (22)$$

where,

$$f(\omega(r+t)) \equiv \int_0^{r+t} (2\psi^2\psi'' + 8\psi\psi'^2) dy. \quad (23)$$

Notice, that since $\psi(wy)$ is a bounded function of amplitude ~ 1 and characteristic frequency ω , we have,

$$f \sim \omega\psi \sim \omega. \quad (24)$$

Thus, we obtain the following relation between the initial free-wave and the leading perturbation due to scattering,

$$\phi_1 \sim \left(\frac{r_*}{r}\right)^3 \phi_0, \quad (25)$$

where,

$$r_* \equiv L_*(\omega L_*)^{\frac{1}{3}}. \quad (26)$$

This equation tells us, that the scattering of Goldstones starts already at a distance r_* , which grows at large ω , as suggested originally in [1]. That is, the classical interaction range exceeds all the quantum length-scales in the problem. This is the signal for formation of a classical configuration.

4 De-Classicalization by Weakly-Coupled UV-completion

We shall now illustrate how weakly-coupled UV-completion de-classicalizes the above theory. Example of such UV-completion is given by an embedding of ϕ in a theory of a weakly-coupled complex scalar

$$\Phi \equiv \frac{1}{\sqrt{2}}(v + \rho)e^{i\phi/v} \quad (27)$$

that spontaneously breaks a global $U(1)$ -symmetry, $\phi \rightarrow \phi + c$, through its vacuum expectation value $\langle \Phi^* \Phi \rangle = \frac{1}{2}v^2$. Generalization to higher symmetries is straightforward. Here ϕ and ρ are angular (Nambu-Goldstone) and radial (Higgs) degrees of freedom respectively.

The microscopic theory has the following Lagrangian,

$$\mathcal{L} = |\partial_\mu \Phi|^2 - \frac{\lambda^2}{8} (2|\Phi|^2 - v^2)^2. \quad (28)$$

Or written in terms of Goldstone and Higgs degrees of freedom,

$$\mathcal{L} = \frac{1}{2} (1 + 2\rho v^{-1} + (\rho v^{-1})^2) (\partial_\mu \phi)^2 + \frac{1}{2} (\partial_\mu \rho)^2 - \left(\frac{m^2}{2} \rho^2 + \frac{\lambda}{2} m \rho^3 + \frac{\lambda^2}{8} \rho^4 \right). \quad (29)$$

This theory describes a massless Goldstone field ϕ , coupled to a radial mode ρ of mass $m \equiv \lambda v$.

Obviously, this theory is a weakly-coupled theory and as such does not classicalize. The r_* -radius at high energies shrinks as $r_* \sim \lambda/\omega$ [2]. We now wish to understand this de-classicalization from the point of view of the low-energy theory and show that it can be understood as an effect of integrating-in in theory (5) a weakly-coupled radial mode ρ .

For this we shall integrate out the weakly-coupled heavy degree of freedom, ρ , and write down an effective interaction for ϕ . We shall work with equations of motion which take the following form,

$$(\square + m^2)\rho = (v^{-1} + \rho v^{-1}) (\partial_\mu \phi)^2 - \frac{3}{2} \lambda m \rho^2 - \frac{\lambda^2}{2} \rho^3, \quad (30)$$

$$\partial^\mu (\partial_\mu \phi (1 + 2\rho v^{-1} + (\rho v^{-1})^2)) = 0. \quad (31)$$

Integrating out ρ to the leading order in $\frac{v^{-2}}{\square + m^2} (\partial_\mu \phi)^2$, gives the following effective equation of motion for ϕ .

$$\partial^\mu \left(\partial_\mu \phi \left(1 + 2 \frac{L_*^4 m^2}{\square + m^2} (\partial_\mu \phi)^2 \right) \right) = 0. \quad (32)$$

where $L_*^2 \equiv (mv)^{-1}$. As we see, this equation of motion differs from (18) by the replacement

$$L_*^4 \rightarrow \frac{L_*^4 m^2}{m^2 + \square}, \quad (33)$$

which is equivalent to (12). In view of this, it is not surprising that the corrected effective theory no longer classicalizes. We shall demonstrate this explicitly. In order to see this, we shall consider the same scattering problem as above, preparing ϕ_0 in form of a collapsing wave of small occupation number and a high center of mass energy $\sqrt{s} = \omega$, and solve for the scattered wave ϕ_1 in the leading order. The equation for ϕ_1 now gets corrected by a massive propagator and after taking into the account (8) can be written as,

$$\square \phi_1 = -2L_*^2 (\partial_\mu \phi_0) \frac{1}{m^2 + \square} \partial^\mu (\partial_\nu \phi_0)^2. \quad (34)$$

Since in the absence of weakly-coupled UV-completion, the classicalization is taking place at distances for which $\square \phi_0^2 \sim L_*^4$, and since $m \ll L_*^{-1}$, for the distances of interest, we can ignore m^2 in the propagator, and the equation simplifies to,

$$\square \phi_1 = -2L_*^2 (\partial_\mu \phi_0) \frac{1}{\square} \partial^\mu (\partial_\nu \phi_0)^2. \quad (35)$$

The latter equation can be easily integrated, noting, that any free wave, satisfying $\square\phi_0 = 0$, obeys the following relations,.

$$\square\phi_0^2 = 2(\partial_\mu\phi_0)^2 \quad \square\phi_0^3 = 3(\partial_\mu\phi_0)(\partial_\mu\phi_0^2), \quad (36)$$

which enable to rewrite (36) in the following form,

$$\square\phi_1 = -\frac{L_*^2}{3}\square\phi_0^3. \quad (37)$$

Thus, we arrive to the relation,

$$\phi_1 = -\frac{L_*^2}{3}\phi_0^3, \quad (38)$$

which implies that classicalization never happens, since r_* shrinks below L_* . Of course, for $r \sim L_*$ one has to consider subleading correction, or even better simply work in high-energy formulation in terms of weakly-coupled complex scalar Φ . This formulation of course suggests that r_* shrinks as $r_* \sim \lambda/\omega$, so that it becomes shorter than any relevant quantum scale in the theory. In other words, system de-classicalizes.

5 Generalization to an Arbitrary Theory

Our analysis showing that a weakly-coupled UV-completion de-classicalizes the theory can be easily generalized to an arbitrary theory in which perturbative unitarity is restored by weakly-coupled physics. Indeed, in classicalizing theories an effective would-be perturbative coupling in a given vertex blows up with increasing energy. This blow-up is the key both for violation of perturbative unitarity as well as for classicalization. Any weakly-coupled UV-completion that shuts-off this growth and restores perturbative unitarity must automatically kill classicalization.

For example, consider a generalization of our previous example to other higher order invariants, by adding derivative couplings of the form,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \sum_{n>1} \frac{a_n L_*^{4(n-1)}}{2n} ((\partial_\mu\phi)^2)^n, \quad (39)$$

where a_n are some constant coefficient. For example, the Dirac-Born-Infeld Lagrangian,

$$L_{DBI} = L_*^{-4} \sqrt{1 + L_*^4 (\partial_\mu\phi)^2}, \quad (40)$$

considered as one of the classicalizing examples in [1], corresponds to a particular choice of coefficients a_n .

The equation of motion has the following form,

$$\partial^\mu \left(\partial_\mu\phi \left(1 + \sum_{n>1} a_n L_*^{4(n-1)} ((\partial_\mu\phi)^2)^{n-1} \right) \right) = 0. \quad (41)$$

For an incoming free wave-packet ϕ_0 , the equation describing a scattered wave ϕ_1 for such a theory is

$$\square\phi_1 = -\partial^\mu \left(\partial_\mu \phi_0 \sum_{n>1} a_n L_*^{4(n-1)} ((\partial_\mu \phi_0)^2)^{n-1} \right), \quad (42)$$

which implies that

$$\phi_1 \sim \frac{1}{r} \sum_{n>1} a_n \left(\frac{\omega}{r^3} L_*^4 \right)^{n-1}. \quad (43)$$

This gives the classicalization radius $r_* \sim L_*(L_*\omega)^{1/3}$.

Integration-in of a weakly-coupled physics at scale m is expressed in modification in which each a_n -th term in (41) is replaced by combinations of the form

$$\frac{m^2}{m^2 + \square} \left((\partial_\nu \phi)^2 \frac{m^2}{m^2 + \square} \left((\partial_\nu \phi)^2 \frac{m^2}{m^2 + \square} \dots (\partial_\nu \phi)^2 \frac{m^2}{m^2 + \square} (\partial_\nu \phi)^2 \right) \dots \right). \quad (44)$$

The perturbative restoration of unitarity requires that each $(\partial_\nu \phi)^2$ is accompanied by at least one propagator $\frac{m^2}{m^2 + \square}$, so that the combination $(\frac{m^2}{m^2 + \square} (\partial_\nu \phi)^2)$ appears $n - 1$ -times.

Then at short distances ($\square \gg m^2$), the equation for the scattered wave becomes

$$\square\phi_1 = -\partial^\mu \left(\partial_\mu \phi_0 \sum_{n>1} a_n m^{2(n-1)} L_*^{4(n-1)} \left(\frac{1}{\square} \left((\partial_\nu \phi_0)^2 \frac{1}{\square} \left((\partial_\nu \phi_0)^2 \frac{1}{\square} \left(\dots \frac{1}{\square} (\partial_\nu \phi_0)^2 \right) \dots \right) \right) \right) \right) \quad (45)$$

This seemingly complicated equation is trivially integrated, by using the identity $\square\phi_0^n = n(n-1)\phi_0^{n-2}(\partial_\mu \phi_0)^2$. The result is

$$\phi_1 = \sum_{n>1} a_n m^{2(n-1)} L_*^{4(n-1)} \phi_0^{2n-1}, \quad (46)$$

where all combinatoric factors have been absorbed in redefinition of parameters a_n , whose values are unimportant for our consideration anyway. What is crucial from the above expression is, that r_* no longer grows with ω , but rather shrinks at least to L_* ; The system de-classicalizes.

6 Window of Classicality

We have seen, that for de-classicalization it is necessary that *every* additional square of the derivative in the self-coupling gets accompanied by a regulating factor (12), which suppresses the derivative-dependence at high momenta and switches off the energy self-sourcing. That is, every interaction term containing $2(n+1)$ powers of

derivatives, must contain at least n massive pole insertions. Schematically, we can write,

$$\partial^{2(n+1)} \rightarrow \partial^{2(n+1)} \left(\frac{m^2}{m^2 + \square} \right)^n, \quad (47)$$

where only the derivatives contributing to the equation of motion count. We have seen, that when $m \ll L_*^{-1}$, this de-classicalization can be understood as a consequence of weakly-coupled UV-completion at the scale m .

What is the meaning of the case $m \gg L_*^{-1}$?

We shall now investigate this limit, and show that in such a case system classicalizes within a finite energy window, which we shall refer to as the *window of classicality*.

For definiteness, let us again consider a scattering problem in a theory governed by the following equation of motion,

$$\partial^\mu \left(\partial_\mu \phi \left(1 + 2L_*^4 \frac{m^2}{\square + m^2} (\partial_\mu \phi)^2 \right) \right) = 0. \quad (48)$$

We shall again consider an initial state to be a free collapsing spherical wave-packet ϕ_0 , of center of mass energy ω . From our previous analysis we know, that in $m \rightarrow \infty$ limit scattering classicalizes for arbitrary $\omega \gg L_*^{-1}$, with the classicalization radius $r_*(\omega)$ being given by (26). However, for finite (but large) $m \gg L_*^{-1}$ the scattering ceases to classicalize above certain threshold energy $\bar{\omega}$. We shall find this critical energy from the following consideration. For a free-wave ϕ_0 the derivative self-interaction in (48) gets diminished when the contribution from the \square operator in the denominator overpowers m^2 . For a free collapsing spherical wave we have the following relation $\square \phi_0^2 \sim \frac{\omega}{r} \phi_0^2$. Thus, for a given ω propagator becomes dominated by \square at distances,

$$r \ll \bar{r}(\omega) \equiv \frac{\omega}{m^2}. \quad (49)$$

We shall refer to $\bar{r}(\omega)$ as the *de-classicalization* radius. System classicalizes as long as

$$\bar{r}(\omega) \ll r_*(\omega), \quad (50)$$

and never in the opposite case. Notice, that in contrast to r_* , the de-classicalization radius \bar{r} is a *quantum* length, which diminishes in $\hbar \rightarrow 0$ limit. Indeed, restoring the powers of \hbar in (49), we get,

$$\bar{r}(\omega) \equiv \hbar^2 \frac{\omega}{m^2}. \quad (51)$$

So any system for which $\bar{r}(\omega) \gg r_*(\omega)$ is quantum, and cannot classicalize. Thus, the equality

$$\bar{r}(\bar{\omega}) = r_*(\bar{\omega}), \quad (52)$$

defines the upper bound of the classicality window,

$$L_*^{-1} < \omega < \bar{\omega}. \quad (53)$$

A precise value of $\bar{\omega}$ is model-dependent, but for any system classicality window exist only if (53) can be satisfied for some $\bar{\omega} > L_*^{-1}$. In the example (48), taking into account (26), we have

$$\bar{\omega} = L_*^2 m^3. \quad (54)$$

At energies above $\bar{\omega}$ theory stops to classicalize, and becomes again a quantum theory. But, quantum theory of what? Below we shall try to give some insight for addressing this question.

7 Beyond the Classicality window: Quantum Theory of Big Objects?

At the level of our analysis, the classicalizing theories that in our parametrization correspond to $m \rightarrow \infty$ (equivalently $\bar{\omega} \rightarrow \infty$) limit, make a consistent physical sense. Or, to say it softer, at least at the level of our analysis, we cannot identify anything obviously-inconsistent about such theories. Their physical meaning can be described as follows.

At low energies, $\omega \ll L_*^{-1}$, we deal with a quantum field theory of a weakly-coupled propagating quantum degree of freedom, ϕ . At energies above L_* , theory enters a classical regime. Scattering takes place at a macroscopic distance r_* that exceeds all the quantum length-scales in the problem (such as L_*). This signals that theory classicalizes. That is, high-energy physics is dominated by extended classical field configurations of ϕ . As any classical state, these can be viewed as composites of many ϕ -quanta in a coherent superpositions. Because of energy self-sourcing, at higher energies these objects become more and more extended and thus probe larger and larger distances. This tendency continues to arbitrarily high energies.

Let us now consider the case of finite m (and thus, finite $\bar{\omega}$). From the first glance, a puzzling thing about this case is, that theory must become again quantum above the scale $\bar{\omega}$. However, quantum theory requires existence of corresponding quantum degrees of freedom. By all the accounts the role of such degrees of freedom cannot be played by ϕ -quanta. This can be understood by matching some of the properties (such as scattering amplitudes) of microscopic and macroscopic theories at the scale $\bar{\omega}$. Such matching tells us, that above the scale $\bar{\omega}$, the role of propagating quantum degrees of freedom cannot be played by ϕ , since an effective "perturbative" coupling of such a degree of freedom would be strong, enhanced by powers of $(L_*\bar{\omega})^2$. Thus, if a sensible theory above $\bar{\omega}$ exists, it must be a quantum theory of something else.

We would like to suggest one such possibility. The hint comes from thinking about the properties of classicalon configurations as of functions of a continuous parameter ω . For values of ω within the classicality window, classicalon configurations

(by default) are characterized by the size $r_*(\omega)$ and energy ω . Within this window their dynamics is governed by classical physics, and quantum influence is negligible. What happens with classicalons above the classicality window? The fact that theory de-classicalizes means that classicalons of mass above $\bar{\omega}$ can no longer exist. In other words, even if we try to prepare such configurations, they should become so quantum-mechanically-unstable, that it should not make sense to talk about them as of well-defined states. In other words, life-time of (would-be) classicalons must become shorter than their size. In order to understand better what picture we have in mind, we shall develop the following analogy.

We shall speculate, that perhaps a QCD-type theory can be considered as a limiting case of such a situation. For this, let us first consider a confining QCD-type theory in which quark masses, m_q , are higher than the QCD scale, Λ_{QCD} . As we know, because of confinement such a theory contains extended objects, the QCD electric flux tubes. The flux-tubes can be either closed or end on heavy quarks, but otherwise they cannot terminate. A long QCD-tube, can break quantum-mechanically by nucleating quark-anti-quark pairs. But, since quarks are heavy, the process is exponentially-suppressed. Nevertheless, the non-vanishing probability of breaking limits the possible size of classical strings and creates a scale somewhat analogous to the upper bound of classicality window, $\bar{\omega}$.

The rate of string-breaking can be estimated, e.g., by treating the quark-anti-quark production as a Schwinger pair-creation process [3] in one-dimensional electric field of the string, or by dualizing decay of unstable cosmic strings (magnetic flux-tubes) via creating monopole-anti-monopole pairs [4]. The resulting probability of breaking per unit length per unit time is given by,

$$\Gamma \sim \Lambda_{QCD}^2 e^{-c(m_q/\Lambda_{QCD})^2}, \quad (55)$$

where c is a model-dependent numerical factor of order one. The above rate can be estimated as follows. By removing a portion of string of length L the energy increment is

$$\Delta E \sim 2m_q - \Lambda_{QCD}^2 L, \quad (56)$$

where the first term is the positive price that comes from nucleating quark-anti-quark pair, whereas the second term is the gain in energy due to removing the portion of a string of length L and tension Λ_{QCD}^2 . Thus, the size of a critical "bubble" that one has to nucleate is given by $L_c \sim m_q/\Lambda_{QCD}^2$ and the Euclidean action on a bounce is $S_E \sim (m_q/\Lambda_{QCD})^2$. Thus, the lifetime of a flux tube of the length $L \gg \Lambda_{QCD}^{-1}$ is

$$\tau_L \sim \frac{1}{\Lambda_{QCD}^2 L} e^{c(m_q/\Lambda_{QCD})^2}. \quad (57)$$

As long as this quantum lifetime is much longer than the string length, L , string can be considered to be a well-defined classical object. Once the lifetime becomes shorter than L , the QCD string ceases to be a well-defined classical state and its

dynamics is governed by quantum mechanics. Thus, the classicality condition is $\tau_L \gg L$. This condition defines a critical length scale,

$$\bar{L} \equiv \Lambda_{QCD}^{-1} e^{\frac{\varepsilon}{2}(m_q/\Lambda_{QCD})^2}, \quad (58)$$

beyond which the classical strings cease to exist. The corresponding string mass,

$$\bar{\omega} = \bar{L} \Lambda_{QCD}^2, \quad (59)$$

sets an upper bound on classicality window (53), which in this case becomes,

$$\Lambda_{QCD} < \omega < \bar{\omega} = \bar{L} \Lambda_{QCD}^2. \quad (60)$$

In order to understand how far the analogy goes, let us consider a high-energy scattering process in such a theory. Because of confinement, at high energies we can distinguish two different types of observers. The first is a short-distance observer that resolves physics at distances shorter than the QCD length, $L_{QCD} \equiv \Lambda_{QCD}^{-1}$. This observer describes the collision process in terms of quarks and gluons and can probe arbitrarily short distances. The second one is a long-distance observer that can probe distances $\gg L_{QCD}$. The latter observer describes scattering process in terms of QCD-flux tubes (glueballs and hadrons). In order to connect the two observations, one has to go through the process of "hardonization" that takes place at distance L_{QCD} . However, the long-distance observer can in principle integrate out this complicated dynamics and describe the scattering process in terms of an effective theory of interacting QCD flux-tubes. This description then should be almost classical within the energy window given by (60) and the corresponding distances $L_{QCD} < L < \bar{L}$.

Of course, as we noted, the strings can break by quark-anti-quark production, but for the strings within the above window this process is negligible. Also, even without quark production, vibrating long strings can decay into smaller loops. First, an oscillating string loop can shorten and quantum-mechanically radiate smaller loops (glueballs). Secondly, a vibrating long string can chop off smaller loops every time the flux-tube intersects with itself. The latter process is very similar to inter-commutation of oscillating cosmic string loops [5] and is described by similar dynamics.

In the language of QCD resonances both processes are seen as decay into lighter glueballs. However, for a flux tube of length $L \gg L_{QCD}$ and curvature radius $\sim L$, the decay into very small loops, of size $\ll L$, is suppressed. Moreover, a characteristic vibration period of such a string is $\sim L$, and therefore the lifetime is at least as long as L . So dynamics of such a macroscopic string-loop is well-approximated by classical physics at least on the time-scale $\sim L$.

The most important fact about the above analogy is the finiteness of classicality window. Since no well-defined classical strings exist with mass above $\bar{\omega}$ and length above \bar{L} , the long distance observer at energies $\omega \gg \bar{\omega}$ is forced back to quantum description and the effective theory at those distances is a theory of breakable flux-tubes.

Let us now carefully go through the valid analogies as well as their limitations.

7.1 Analogy

We tried to establish a certain analogy between the theories with finite classicality window and QCD-type theory with breakable flux tubes. The essential property of any former theory is the existence of at least three length-scales. Two of them, L_* and \bar{r} are quantum in nature and vanish in the limit $\hbar \rightarrow 0$. Whereas the third one r_* is classical and independent of \hbar . Another essential property is that two of the scales, $\bar{r}(\omega)$ and $r_*(\omega)$ are energy-dependent. The classicality window exist if for some interval of energies (53), the scale $r_*(\omega)$ can be a dominant length scale. For such energy the classicalon configuration is formed before the system has any chance to probe the quantum length-scales \bar{r} or L_* .

In case of QCD flux tubes, the roles of the scales L_* , r_* and \bar{r} are played by the QCD length L_{QCD} , the string length L , and the ratio of the latter length-squared to the critical length (L^2/\bar{L}), respectively. In other words, the dictionary is:

$$L_* \rightarrow L_{QCD} \tag{61}$$

$$r_* \rightarrow L \tag{62}$$

$$\bar{r} \rightarrow \frac{L^2}{\bar{L}} \tag{63}$$

The physical meaning of the above correspondence is clear. Obviously the QCD-length, L_{QCD} , plays the same role as L_* , since it marks the length scale beyond which classical flux tubes exist. Both of the scales are quantum in nature. The string length, L , is an obvious classical scale in the problem. We can talk about classical strings only as long as $L > L_{QCD}$. Finally, it is obvious that the ratio L^2/\bar{L} controls classicality of strings, since strings cease to exist when $\bar{L} < L$.

7.2 Limitations: Light-quark QCD as a Collapsed Classicality Window?

The above analogy has to be taken with an extreme care, as it does have obvious limitations. At the current level of the analysis it only serves as an existence proof of a situation when a classical behavior within a limited interval of energies is possible. It shows, that we can have a well-defined microscopic theory for which the effective high-energy/long-distance behavior can be understood as interpolation between quantum to classical and back to quantum regimes. The very last transition being triggered by quantum instability of the extended (would-be) classical objects that sets-in above their critical size and energy.

On the other hand there are clear differences. For example, in high-energy QCD we can scatter heavy quarks at arbitrarily short distances. In this respect, the roles of r_* and L are very different. In the example (48) the scattering does happen at distance r_* , whereas in QCD the long tubes are an outcome of a short-distance scattering process through hadronization.

This difference has to do with the qualitative differences between the low energy spectra of the two theories. The model (48) at low energies, $\omega \ll L_*^{-1}$, includes a light propagating degree of freedom, a Goldstone boson ϕ . Whereas, QCD without light quarks is a theory with the mass gap, with no propagating degrees of freedom in deep infrared $\omega \ll \Lambda_{QCD}$. Because of this, classicalization of QCD flux tubes does not represent an UV-completion of any low-energy Goldstone-type theory.

We could have tried to make the analogy closer by taking the limit of light-quarks, $m_q \ll \Lambda_{QCD}$. Of course, in such a case there is a well-defined low-energy description in terms of a pion chiral Lagrangian, but now classicalization is spoiled from the other end, since QCD flux tubes simply become unstable. All the three length-scales, classical and quantum, become of the same order,

$$L_{QCD} \sim L \sim \tau_L. \quad (64)$$

The classicality window collapses to a point!

We thus see, that at least in ordinary QCD, there is no parameter choice for which one could have a low energy pion theory and simultaneously maintain a significant classicality window.

This consideration indicates that in certain crude (but well-defined) sense, ordinary QCD can be thought of as a limit of would-be classicalizing theory in which the classicality window collapsed to a single scale, Λ_{QCD} . Whether the above-presented view sheds any useful light at the QCD-confinement dynamics, is unclear, but it is certainly illustrative for understanding regimes of classicalization phenomenon.

8 conclusions

Many weakly coupled microscopic theories, such as models with spontaneously broken global symmetries, Higgsed gauge theories and high-energy QCD, are known to result at low energies in derivatively interacting massless or light Nambu-Goldstone type degrees of freedom. On the other hand, it was argued recently [1, 2], that such theories have a tendency to classicalize in very high energy scatterings. Namely, the theories in question contain a classical length-scale r_* that governs the range of interaction and grows with increasing center of mass energy in such a way that exceeds all the quantum length-scales in the problem. Classicalization is therefore taking place at distances much larger than the Compton wave-length of any weakly coupled physics that would-be responsible for maintaining the perturbative unitarity at short distances.

This behavior creates a seeming puzzle, since a long-distance theory should not know about the short-distance UV-completion and thus, should continue to classicalize even when the short distance interactions are UV-completed by weakly-coupled physics that maintains perturbative unitarity. But, how can these two dramatically different behaviors of the scattering amplitudes be reconciled?

In the present note we have resolved this puzzle, and have shown that the two behaviors are not reconciled, and the theory can chose only one way: Classicalize

or not classicalize. Integration-in of any weakly-coupled short distance physics that maintains perturbative unitarity automatically kills classicalization: r_* -radius shrinks below the quantum-mechanical length-scales and becomes irrelevant; system de-classicalizes.

The reason why we arrived to a seeming puzzle to start with was the wrong intuition, that the high-energy structure of the theory should not affect its long-distance behavior. In classicalizing theories this intuition breaks down, since high-energy physics probes long distances, due to existence of the extended classical objects. In other words, in classicalizing theories very high energies are not equivalent to short distances but rather to very large distances. As a result, the energy densities corresponding to r_* are absolutely enough for theory to “know” whether it classicalizes or not. This is why a perturbative-unitarity-restoring weakly-coupled physics with mass m below L_*^{-1} ruins classicalization, which, by the way, in this case is no longer needed. On the other hand, if we deprive a theory of a weakly-coupled UV-completion by pushing $m \rightarrow \infty$, theory puts up a “self-defense” and unitarizes by classicalization.

We have shown, that interpolation between the two regimes can be parameterized by defining a new quantum radius \bar{r} , which we called the radius of de-classicalization. System de-classicalizes when there is no choice of center of mass energy for which the classical radius r_* could exceed the quantum scale \bar{r} , and classicalizes in the opposite case. The former situation takes place when $m < L_*^{-1}$, whereas the latter, when $m = \infty$.

This consideration lead us to a logical possibility for the existence of an intermediate regime, in which r_* dominates over \bar{r} only within a finite energy window, called window of classicality. This phenomenon takes place when m is large but finite, $m \gg L_*$. However, in this case m can no longer be identified with a mass scale of new weakly-coupled quanta and question arises about its physical meaning. In other words, finiteness of the classicality window rises the question about the physical meaning of theory beyond this window. We have given an evidence that above the classicality window we deal with a theory of unstable extended objects.

In order to support this intuition, we have established an analogy with QCD-type theory that contains such objects, the unstable QCD flux tubes. However, for flux tubes of size $\gg L_{QCD}$ to exist, quarks have to be heavier than the QCD scale. But, in this case no low energy analog of Goldstone-type classicalizer field ϕ exists. Creating such an analog, by taking the light quark limit, automatically makes flux-tubes unstable and collapses the classicality window to a point.

This fact sheds for us two different lights. First, it provides for us an alternative language, in terms of QCD flux tubes, for explaining why theory of real QCD pions does not classicalize, despite the presence of derivative self-interactions. Secondly, it tells us that real QCD can be viewed as a theory with a collapsed classicality window.

Finally it would be interesting to explore a possible connection (if any) between some of the presented ideas about classicalization and recently suggested possible

UV-finiteness of 5D super-Young-Mills [6].

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Note Added

Before submitting this paper, a paper by Heisenberg [7] from 1952 was brought to our attention ¹. In this paper Heisenberg tries to account for high-multiplicity in strong interaction scattering by studying propagation of shock-waves in pion field, and argues that derivative self-interactions are important for the correct description of the scattering. Since QCD-type theories only serve as an useful rough analogy, rather than the main focus of our work, the phenomenological validity of Heisenberg's result is secondary for this comment. What is remarkable for us is his appreciation of importance of the derivative self-couplings for creating multiplicity of states in high-energy scattering. This emphases resonates with our ideas on classicalization.

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